Math 43 Midterm 3 Review Solutions

In addition to the following review questions, you must be able to solve any of the questions from the 3D Lines & Planes handout.

[1] [a] $<-2,-4>\cdot<x,y>=0 \quad\Rightarrow\quad -2x-4y=0$

Let $x=2$ and $y=-1$

$\dfrac{1}{\|<2,-1>\|}<2,-1> \;=\; \frac{1}{\sqrt{5}}<2,-1> \;=\; <\frac{2}{\sqrt{5}},-\frac{1}{\sqrt{5}}>$

[b] $-2<0 \quad\Rightarrow\quad \theta=\pi+\tan^{-1}\frac{-4}{-2}\approx 4.25$ radians

[c] $2<8\cos\frac{2\pi}{3},8\sin\frac{2\pi}{3}>-<-2,-4> \;=\; 2<-4,4\sqrt{3}>-<-2,-4> \;=\; <-8,8\sqrt{3}>-<-2,-4>$

$= \;<-6,4+8\sqrt{3}> \;=\; -6\vec{i}+(4+8\sqrt{3})\vec{j}$

[2] [a] $\cos^{-1}\dfrac{<0,2,-3>\cdot<-1,-3,4>}{\|<0,2,-3>\|\,\|<-1,-3,4>\|} \;=\; \cos^{-1}\dfrac{-18}{\sqrt{13}\sqrt{26}} \;\approx\; 2.94$ radians

[b] $<0,2,-3>\times<-1,-3,4>=<-1,3,2>$

$\dfrac{1}{\|<-1,3,2>\|}<-1,3,2> \;=\; \frac{1}{\sqrt{14}}<-1,3,2> \;=\; <-\frac{1}{\sqrt{14}},\frac{3}{\sqrt{14}},\frac{2}{\sqrt{14}}>$

[c] $proj_{<-1,-3,4>}<0,2,-3> \;=\; \dfrac{<0,2,-3>\cdot<-1,-3,4>}{<-1,-3,4>\cdot<-1,-3,4>}<-1,-3,4> \;=\; \dfrac{-18}{26}<-1,-3,4>$

$= \; -\frac{9}{13}<-1,-3,4> \;=\; <\frac{9}{13},\frac{27}{13},-\frac{36}{13}>$

$<0,2,-3>-<\frac{9}{13},\frac{27}{13},-\frac{36}{13}> \;=\; <-\frac{9}{13},-\frac{1}{13},-\frac{3}{13}>$

$<0,2,-3> \;=\; <\frac{9}{13},\frac{27}{13},-\frac{36}{13}>+<-\frac{9}{13},-\frac{1}{13},-\frac{3}{13}>$

[d] $<-7-x,4-y,-8-z>=<-1,-3,4>$

$-7-x=-1,\quad 4-y=-3,\quad -8-z=4 \quad\Rightarrow\quad (x,y,z)=(-6,7,-12)$

[e] $<a,b,-5>=m<-1,-3,4> \quad\Rightarrow\quad a=-m,\quad b=-3m,\quad -5=4m \quad\Rightarrow\quad m=-\frac{5}{4},\quad a=\frac{5}{4},\quad b=\frac{15}{4}$

[f] $<7,c,-5>\cdot<-1,-3,4>=0 \quad\Rightarrow\quad -7-3c-20=0 \quad\Rightarrow\quad c=-9$

[3] [a] octant $2+4=6$

[b] $(-5-4,-2+6,3-2)=(-9,4,1)$

[c] $<3--3,2-4,-1--2>=<6,-2,1>$

[d] $<-3--5,4--2,-2-3>=<2,6,-5>=2\vec{i}+6\vec{j}-5\vec{k}$

[e] $\sqrt{4+36+25}=\sqrt{65}$

[f] $-\frac{1}{\sqrt{65}}<2,6,-5>=<-\frac{2}{\sqrt{65}},-\frac{6}{\sqrt{65}},\frac{5}{\sqrt{65}}>$

[g] $\frac{6}{\sqrt{41}}<6,-2,1>=<\frac{36}{\sqrt{41}},-\frac{12}{\sqrt{41}},\frac{6}{\sqrt{41}}>$

[h] $(\sqrt{41})(3)\cos 2\approx -7.99$

[i] $(\sqrt{41})(3)\sin 2\approx 17.47$

[j] $\overrightarrow{RQ}\times\overrightarrow{RP}=<6,-2,1>\times<-2,-6,5>=<-4,-32,-40>=-4<1,8,10>$

$\frac{1}{2}\|-4<1,8,10>\|=2\|<1,8,10>\|=2\sqrt{165}$

[k] $\cos^{-1}\dfrac{<6,-2,1>\cdot<-2,-6,5>}{\|<6,-2,1>\|\,\|<-2,-6,5>\|} \;=\; \cos^{-1}\dfrac{5}{\sqrt{41}\sqrt{65}} \;\approx\; 1.47$ radians

[l] $<4,0,-5>\cdot<-5-3,-2-2,3--1>=<4,0,-5>\cdot<-8,-4,4>=-52$

[m] normal vector $=\overrightarrow{RQ}\times\overrightarrow{RP}=<-4,-32,-40>$ or $<1,8,10>$

$1(x+5)+8(y+2)+10(z-3)=0 \quad\Rightarrow\quad x+8y+10z-9=0$

[n] $x=-5+6t,\quad y=-2-2t,\quad z=3+t$

[o] direction vector $= <-2, -3, 1>$ or $< 2, 3, -1 >$

$$\frac{x-3}{2} = \frac{y-2}{3} = \frac{z+1}{-1} \text{ or } -z-1$$

[p] center $= (\frac{-5+3}{2}, \frac{-2+2}{2}, \frac{3+-1}{2}) = (-1, 0, 1)$

radius $^2 = (3--1)^2 + (2-0)^2 + (-1-1)^2 = 24$

$(x+1)^2 + y^2 + (z-1)^2 = 24$

[4] $x<0,\ y>0,\ z>0 \quad \Rightarrow \quad$ octant 2

$x<0,\ y<0,\ z>0 \quad \Rightarrow \quad$ octant 3

$x>0,\ y>0,\ z<0 \quad \Rightarrow \quad$ octant $1+4=5$

$x>0,\ y<0,\ z<0 \quad \Rightarrow \quad$ octant $4+4=8$

[5] [a] $x^2 - 4x + y^2 + 6y + z^2 + 10z = -29$

$(x^2 - 4x + 4) + (y^2 + 6y + 9) + (z^2 + 10z + 25) = -29 + 4 + 9 + 25$

$(x-2)^2 + (y+3)^2 + (z+5)^2 = 9$

center $= (2, -3, -5)$

radius $= 3$

[b] $(x-2)^2 + (y+3)^2 + (0+5)^2 = 9 \quad \Rightarrow \quad (x-2)^2 + (y+3)^2 = -16$

no $xy-$trace

$(x-2)^2 + (0+3)^2 + (z+5)^2 = 9 \quad \Rightarrow \quad (x-2)^2 + (z+5)^2 = 0$

$xz-$trace is point $(2, 0, -5)$

$(0-2)^2 + (y+3)^2 + (z+5)^2 = 9 \quad \Rightarrow \quad (y+3)^2 + (z+5)^2 = 5$

$yz-$trace is circle in $yz-$plane, center $= (0, -3, -5)$, radius $= \sqrt{5}$